AMS Foundation Exam - Part A, January 2020

Name: ________________________________ ID Num. ____________________

Part A: ______ / 75

Part B: ______ / 75 Total: ______ / 150

This component of the exam (Part A) consists of two sections (Linear Algebra and Advanced Calculus) with four problems in each. Each question is worth 25 points; choose **THREE** questions to answer from **EACH** section. Each problem should be solvable in approximately 20 minutes or less. Provide your answer in the space provided, and show all work. If extra sheets are used, place them inside the booklet and note on the cover page how many additional pages are included.

Good Luck!

Section 1: Linear Algebra

Choose three of the four problems to solve.

1. Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by

$$F(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t).$$

   (a) Show that F is linear. (b) Find a basis and the dimension of $\mathrm{Im}(F)$ and $\mathrm{Ker}(F)$.

2. Let S and T be bases for a two-dimensional vector space V and let A and B be operators on V. Suppose that

$$[A]_S = \begin{bmatrix} 5 & c \\ c & -1 \end{bmatrix} \text{ and } [B]_T = \begin{bmatrix} -3 & 0 \\ 0 & 7 \end{bmatrix}.$$

Determine all values for c such that $A = B$.

3. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$. Prove that $\mathrm{Ker}(\mathbf{A}) = \mathrm{Im}(\mathbf{A}^T)^\perp$.

4. Prove that $V = U \oplus W$ if and only if $V = U + W$ and $U \cap W = \{0\}$.

Section 2: Advanced Calculus

Choose three of the four problems to solve.

1. Let $c \neq 0$ and suppose $\lim_{x \to c} f(x) = L$. Show that $\lim_{x \to \frac{1}{c}} f\left(\frac{1}{x}\right) = L$.

2. Let $a > 0$. Evaluate $I(a) = \int_0^\infty \mathrm{d}x \, \frac{e^{-ax^2} - e^{-x^2}}{x}$.

3. Find the highest point (largest z) on the curve of intersection of the surfaces $x^2 + y^2 + z^2 = 36$ and $2x + y - z = 2$.

4. Let $a, b, c > 0$. Evaluate $\iiint_{\mathbb{R}^3} \mathrm{d}V\, e^{-(ax^2+by^2+cz^2)}$.